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# A SIMPLE FORMULATION FOR COMPLEX MODES OF IMAGE LINES

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## Abstract

We present here an analysis of a shielded image line by the simple method of effective dielectric constant. The concept of negative effective dielectric constant is utilized for a below-cutoff mode, so that the occurrence of backward flow of energy and the existence of complex modes in such a class of waveguides become physcially transparent via the familiar physical phenomena associated with the class of dielectric-plasma structures. The merits of this method is its simplicity and general applicability to a large class of structures. Numerical results are shown to illustrate the accuracy obtainable from the present approach.

## Introduction

Complex modes were first shown to exist in a dielectric-loaded circular waveguide by Clarricoats and Slinn [1], and were correctly attributed to the contra-flow coupling between forward and backward waves. Mathematically, these complex modes form an integral part of a complete set of waveguide modes and they can be crucial to the analysis of wave scattering by discontinuities in a waveguide. Because of their unusual nature, many attempts had been made for a better understanding of the complex modes. Strube and Arndt [2] related the complex modes to the leaky phenomenon, and Tzuang and Lin [3] interpreted them in terms of mode coupling.

While the previous analyses have been focused on various mathematical methods to characterize complex modes, they have to rely on ellaborate numerical procedures. In this paper, we present a simple approach to the analysis of complex modes in a rectangular dielectric-laoded waveguide, such as a shielded image line [2]. First, we employ the method of effective dielectric constant (EDC) [4-6], which replaces a shielded image line by an equivalent structure, i. e., a parallel-plate waveguide partially filled with a dielectric slab. Second, we utilize the concept of negative effective dielectric constant for below-cutoff modes in each constituent region of the waveguide. By taking advantage of well-known physical phenomena associated with dielectric-plasma structures, the prediction and explanation of complex modes become physically transparent. This method permits us to predict the occurence of complex modes and is generally applicable to a large class of structures. Numerical results are given to illustrate the effectiveness and simplicity of the present approach. In short, the merits of this work are: (1) mathematical simplicity and effectiveness, (2) physical insight to provide a better understanding of the wave processes involved in the shielded image line, and (3) practical usefulness for a preliminary design of shielded dielectric waveguides.



## Method of Analysis

The crosssectional geometry of a shielded image line is shown in Fig. 1(a). To employ the EDC method, the first step is to divide such a crosssection into two constituent subregions, as indicated by the dashed line. The second step is to determine the effective dielectric constant of each subregion separately. Finally, the third step is to replace the original waveguide by a planar multilayer structure from which the guiding characteristics of the original waveguide can be analyzed approximately with relative ease. In this case, the two constituent regions of the shielded image line can be regarded as two parallel-plate waveguides: one totally filled with air and the other partially filled with a dielectric layer. Denote the effective dielectric constant of the fundamental mode of the air-filled waveguide by $\varepsilon_{eff}^{(a)}$ and that of the partially filled dielectric waveguide by $\varepsilon_{eff}^{(d)}$; the shielded image line is then replaced by a planar multilayer structure, as shown in Fig. 1(b). As an example, for a structure with the set of parameters: $\varepsilon_d = 20$ $\varepsilon_a = 1$, h = 0.7899 cm and w = 0.345 cm, the



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effective dielectric constants of the two subregions are shown in Fig. 2, for the TE modes with respect to the lateral direction. Between the cutoff frequencies of the two subregions: $f_d$ = 2.4028 GHz and $f_a$ = 9.495 GHz, the two effective dielectric constants of the constituent subregions have opposite signs, and the planar structure has an equivalent of a dielectric-plasma interface. This provides a simple and clear physical picture for the interpretations of complex modes

## Numerical Results

To illustrate the physical phenomena associated with a structure containing a dielectric-plasma interface, we consider first a simple case of $\varepsilon_{eff}^{(a)}$ = -0.1, and $\varepsilon_{eff}^{(d)}$ = 3.0, $t_d$ = 0.7 cm, and $t_a$ = 0.09 cm for the structure shown in Fig. 1(b), The dielectric-plasma interface alone may support a surface plasmon mode for which the effective dielectric constant is: $\varepsilon_{eff}$ = -0.10345. Furthermore, since $t_d$ is much larger the $t_a$, we may regard the structure as being a uniform parallel-plate waveguide with the dielectric medium perturbed by a thin plasma layer. The dispersion curves for the surface plasmon mode and the lowest TM mode with respect to the vertical direction are shown in Fig. 3, as designated herein by SP and $TM_1$, respectively; they may be regarded as the unperturbed dispersion curves of the dielectric-plasma structure. These unperturbed disperion curves are brought up here as a basis for the ensuing physical interpretations; actually, the dispersion curves of the structure can be easily obtained and the results are also shown in the solid line in Fig. 3. Evidently, the complex-mode region occurs in the vicinity of the intersection point of the unperturbed dispersion curves. Thus, the complex-mode phenomenon can be attributed to the interaction or coupling between the unperturbed or basic modes of the waveguide, one forward and the other back propagating. This is consistent with the concept of mode coupling, as previously reported.[3] Specifically, the complex region occurs in the frequency range between about 3 and 8 GHz, exhibiting the real and imaginary parts of the complex propagation constant. It is noted that we have examined the Poynting vector in the equivalent plasma layer with negative dielectric constant; indeed, it is in the opposite direction to that in the ordinary dielectric layer. Thus, the negative dielectric constant of the plasma layer provides the mechanism for the contra-flow of energy in the uniform waveguide.

For the shielded image line analyzed by the EDC method, the dielectric constants of the planar structure, $\varepsilon_{eff}^{(a)}$ and $\varepsilon_{eff}^{(d)}$, are frequency-dependent, but their accurate values can be easily determined, as given in Fig. 2. Based on the knowledge on wave propagation along the dielectric-plasma structures, discussed above, we can expect that the shielded image line may have complex-mode regions, depending on the thichnesses of the two subregions. The dispersion curves of the shielded image line are calculated by the EDC method and also by the exact method of mode matching, for the case of $t_d$ = 0.32 cm. The calculations are carried out for many frequencies and the results of the first two $E^{(y)}$ even modes of the shielded image line are plotted in Fig. 4, showing regions of complex mode. First the accurate results obtained from the rigorous method of mode matching are shown in the solid lines and found to agree very well with those given by Strube and Arndt [2] for the same structure. It is noted that the EDC method predicts and locates the complex-mode region only in the qualitative sense. However, its numerical accuracy may be improved by using a modified effective dielctric constant[7] for the air-filled parallel-plate waveguide, and the simple EDC method can still be employed. The results of the modified EDC method are also included in Fig. 4, as shown in the dashed lines. In comparison, the modified EDC method yields numerical results that agree quite well with the exact ones. This approach provides a solid basis of contra-flow coupling of modes, resulting in the complex modes in the shielded image line. The detailed analysis and further data on will be given in the presentation.

## Conclusion

We have introduced the concept of negative effective dielectric constant in combination with the method of effective dielectric constant for the analysis of dielectric-loaded waveguides. The method is very simple and generally applicable to a large class of structures. A shielded image line is investigated in details, with numerical data to show the effectiveness of the method. It is shown that the present approach is reasonably accurate and should be particularly be useful for the prediction and explanation of the complex modes in a dielectric-loaded waveguide. This provides valuable guidelines for the design shielded image line.

## References

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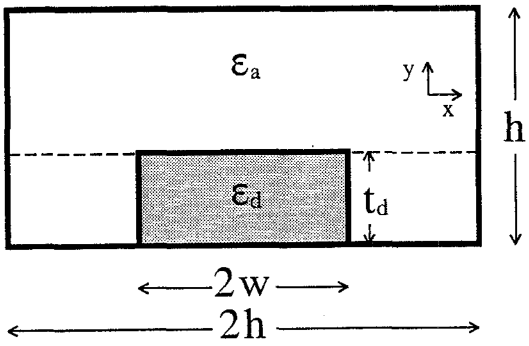
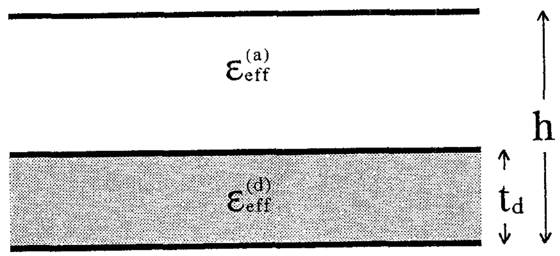



(a) Shielded image line




(b) Parallel-plate waveguide

Fig. 1 Three-dimensional shielded image line and its two dimensional modal

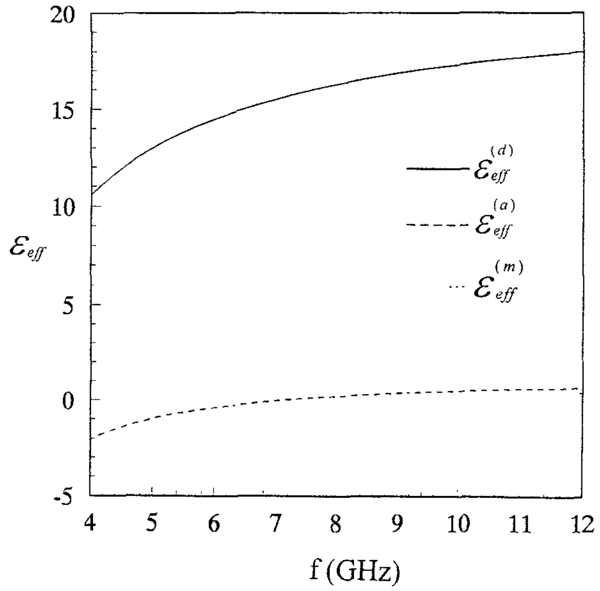



Fig. 2 The effective dielectric constants of the constituent regions.

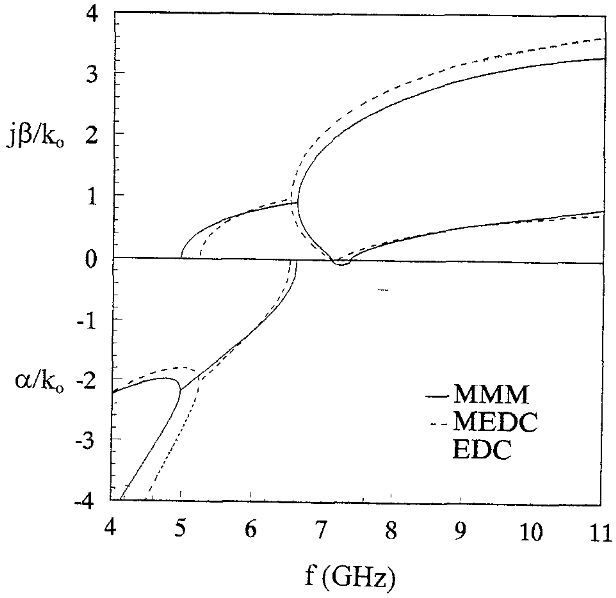



Fig. 4 Comparison of numerical results obtained by three different methods: w = 0.32 cm, $t_d$ = 0.345 cm, h = 0.7899 cm, $\varepsilon_d$ = 20.0, $\varepsilon_a$ = 1.0.

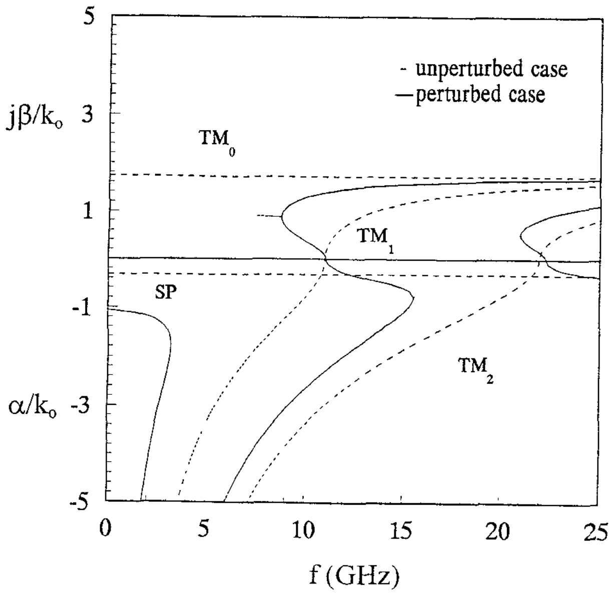



Fig. 3 Dispersion curves for waveguide with thin plasma layer: $t_a$ = 0.09cm, $t_d$ = 0.70cm, $\varepsilon_{eff}^{(a)}$ = 3.0 and $\varepsilon_{eff}^{(d)}$ = -0.1.